

MATH 140A Review: Set builder notation in $\mathbb{R}$

Facts to Know:

A set S in $\mathbb{R}$ is a *collection of real numbers*. For example,

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\} \quad \{-2, 3, 2020, \pi\}$$

$$\{2, 4, 6, 8, \dots\} = \{n \in \mathbb{N} : n \text{ is even}\} \quad [0, 1) = \{x : 0 \leq x < 1\}$$

Notation:

• (belongs) $2020, 6 \in \{2, 4, 6, 8, \dots\}$

• (does not belong) $-6 \notin \{2, 4, 6, 8, \dots\}$

• (subset) $A \subset B \iff \text{if } x \in A, \text{ then } x \in B$

$$A = \{4, 8, 10\} \subset \{2, 4, 6, 8, \dots\} = B$$

• (the empty set) $\emptyset$ is the set with no elements



Example: Simplify the notation:

1. (union)

$$x \in [0, 1) \cup [1/2, 2] \iff x \in [0, 1) \text{ or } x \in [1/2, 2]$$

$$\iff x \in [0, 2]$$



$$[0, 1) \cup [1/2, 2] = [0, 2]$$

2. (intersection)

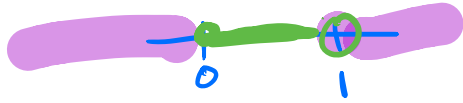
$$x \in [0, 1) \cap [1/2, 2] \iff x \in [0, 1) \text{ and } x \in [1/2, 2]$$

$$\iff x \in [1/2, 1)$$



$$[0, 1) \cap [1/2, 2] = [1/2, 1)$$

3. (complement)



$x \in [0, 1]^c$ iff $x \notin [0, 1]$ and $x \in \mathbb{R}$

iff $x \in (-\infty, 0) \cup [1, \infty)$

$$[0, 1]^c = (-\infty, 0) \cup [1, \infty)$$

Example: Describe the following sets:

1. (integer numbers $\mathbb{Z}$)

$$\begin{aligned}\mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, 3, 4, \dots\} \\ &= \mathbb{N} \cup \{-1, -2, -3, \dots\} \cup \{0\}\end{aligned}$$

2. (rational numbers $\mathbb{Q}$)

$$2, 3, \frac{3}{4}, \frac{1}{2}, -\frac{1}{2}, \in \mathbb{Q}$$

$$\sqrt{2}, \pi \notin \mathbb{Q}$$

$$\mathbb{Q} = \{x \in \mathbb{R} : \exists p \in \mathbb{Z}, \exists q \in \mathbb{N}, x = \frac{p}{q}\}$$

3. (irrational numbers)

x is irrational if and only if $x \notin \mathbb{Q}$

$$\sqrt{2}, \pi \notin \mathbb{Q} \Rightarrow \sqrt{2}, \pi \in \mathbb{Q}^c$$